ISOCURVATURE PERTURBATIONS IN MULTIPLE INFLATIONARY MODELS

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Abstract

Dynamics of long-wave isocurvature perturbations during an inflationary stage in multiple (multi-component) inflationary models is calculated analytically for the case where scalar fields producing this stage interact between themselves through gravity only. This enables to determine correct amplitudes of such perturbations produced by vacuum quantum fluctuations of the scalar fields during the multiple inflationary stage. Exact matching to a post-inflationary evolution that gives the amplitude of isocurvature perturbations in the cold dark matter model with radiation is performed in the case where a massive inflaton field remains uncoupled from usual matter up to the present time. For this model, isocurvature perturbations are smaller than adiabatic ones in the region of the break in the perturbation spectrum which arises due to a transition between the two phases of inflation, but they may be much bigger and have a maximum at much shorter scales. The case of an inflaton with a quartic coupling which remains uncoupled after inflation is considered, too.

1 Introduction

Inflationary cosmological models in which a de Sitter (inflationary) stage is produced by a number of effective scalar fields (inflatons) are called multiple (or multi-component) [1] (see [2] for a general review). A double inflationary model with two scalar fields [3]-[9] is a specific case of them. Note that extended inflationary [10] models or inflationary models in the Brans-Dicke theory of gravity may also be considered as belonging to this class of models after transformation to the Einstein frame. Double inflationary models producing a step-like spectrum of initial adiabatic perturbations give a possibility to reconcile the CDM model with observations without introducing neutrinos [11, 12]. If N is the number of light scalar fields at the inflationary stage in such a theory ($|m_i^2| \ll H^2,\ H \equiv \dot{a}/a$, where $a(t)$ is the scale factor of the FRW isotropic cosmologiocal model), then N independent branches of non-decaying quantum fluctuations of the scalar fields are generated during the inflationary stage similar, and in addition, to quantum fluctuations of gravitons (the resulting energy spectrum of the latter was first correctly calculated in [13]). However, only one linear combination of these fluctuations produces the growing scalar (adiabatic) mode which is usually assumed to be responsible for the formation of galaxies, stars (and other compact objects) and the large-scale structure of the Universe. The other $N-1$ modes are isocurvature fluctuations during the inflationary stage (they were first considered in [14]).

Isocurvature fluctuations are less universal than adiabatic ones. First, they only appear in multiple, not single inflationary models. Second, they might not survive up to the present time (and typically do not). Really, they can exist now only if at least one of the inflaton scalar fields remains non-thermalized and uncoupled from the usual matter (radiation, baryons and leptons) during the whole evolution of the Universe from the inflationary era until the present period (so that the corresponding particles or products of their decay constitute a part of cold dark matter) - a rather strong assumption. Finally, even so, their amplitude (in sharp contrast with adiabatic perturbations) does depend on the form of the transition from inflation to the radiation-dominated FRW stage. Nevertheless, isocurvature perturbations represent an interesting and important object of investigation, especially because some candidates for such inflatons that might survive from the inflationary era up to the present time are already known - dilatons in the

Brans-Dicke theory and superstring induced theories (see [15] for the latter), axions in "natural" inflation [16], etc.

So, in the present paper we consider isocurvature perturbations in the simplest case when N scalar fields have arbitrary self-interaction potentials but interact mutually through gravity only, i.e. the interaction potential $V(\phi_1,\ ...\phi_N) = \sum_1^N V_n(\phi_n)$. The general quantitatively correct expression for adiabatic perturbations generated in this model was obtained in [1]. First, we find the general solution for the behaviour of long-wave isocurvature perturbations during the multiple inflationary stage and then determine the correct coefficients in it by exact matching to vacuum quantum fluctuations of the scalar fields in the approximately de Sitter background during the inflationary stage (Sec. 2). After that, in Sec. 3, we investigate some cases where it is possible to match this solution to the radiation-dominated FRW model with a small admixture of non-thermalized massive particles ("cold dark matter"). The most interesting case with respect to cosmological applications turns out to be the double inflationary model with two massive inflatons, the heavier one remaining uncoupled from usual matter after inflation. The isocurvature perturbation spectrum in this model has a maximum on small scales whose value may be rather large.

2 Behaviour of perturbations during a multiple inflationary stage

We consider the following Lagrangian density describing gravity plus N scalar fields

$$L = -\frac{R}{16\pi G} + \sum_{j=1}^{N}\Big(\frac{1}{2}\phi_{j,\mu}\phi_j^{;\mu} - V_j(\phi_j)\Big) \qquad (1)$$

where $\mu = 0,..,3,\ c = \hbar = 1$ and the Landau-Lifshitz sign conventions are used. Note that the N scalar fields interact only gravitationally. The space-time metric has the form

$$ds^2 = dt^2 - a^2(t)\delta_{mn}dx^m dx^n, \qquad m,n = 1,2,3. \qquad (2)$$

Spatial curvature may always be neglected because it becomes vanishingly small after the first few e-folds of inflation. The homogeneous background is

treated classically, it is determined by the scale factor $a(t)$ and the N scalar fields $\phi_j(t)$. Their equation of motion is given by

$$H^2 = \sum_{j=1}^{N} \frac{8\pi G}{3}\Big(\frac{\dot{\phi}_j^2}{2} + V_j(\phi_j)\Big), \tag{3}$$

$$\ddot{\phi}_j + 3H\dot{\phi}_j + V'(\phi_j) = 0, \qquad\qquad j = 1,..,N \tag{4}$$

where a dot denotes a derivative with respect to t while a prime stands for a derivative with respect to ϕ_j. From (4), we get the useful equation

$$\dot{H} = -4\pi G \sum_{j=1}^{N} \dot{\phi}_j^2. \tag{5}$$

In these models, therefore, H always decreases with time.

Let us turn now to the inhomogeneous perturbations. We consider a perturbed FRW background whose metric, in the longitudinal gauge, is given by

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(t)(1 - 2\Psi)\delta_{mn}dx^m dx^n. \tag{6}$$

We get from the perturbed Einstein equations ($exp(i\mathbf{kr})$ spatial dependence is assumed and the Fourier transform convention is $\Phi(\mathbf{k}) \equiv (2\pi)^{-3/2}\int \Phi(\mathbf{r})e^{-i\mathbf{kr}}d^3\mathbf{k}$)

$$\begin{aligned} \Phi &= \Psi, && (7)\\ \dot{\Phi} + H\Phi &= 4\pi G \sum_{j=1}^{N} \dot{\phi}_j \delta\phi_j, && (8)\\ \delta\ddot{\phi}_j + 3H\delta\dot{\phi}_j + \Big(\frac{k^2}{a^2} + V_j''\Big)\delta\phi_j &= 4\dot{\phi}_j\dot{\Phi} - 2V_j'\Phi, \qquad j = 1,..,N. && (9) \end{aligned}$$

We see that when we have more than one scalar field, the dynamics of the perturbed system cannot be described by just one equation for the master quantity Φ. It is remarkable that without solving the system (7-9) we can immediately write its two exact solutions desribing the adiabatic modes in the formal limit $k \to 0$:

$$\begin{aligned} \Phi &= C_1\left(1 - \frac{H}{a}\int_0^t a dt'\right) + C_2\frac{H}{a}, && (10)\\ \frac{\delta\phi_j}{\dot{\phi}_j} &= \frac{1}{a}\left(C_1\int_0^t a dt' - C_2\right), \qquad j = 1,..,N && (11) \end{aligned}$$

where $a(t)$, $\phi_j(t)$ satisfy the exact background equations (3,4) and C_1 and C_2 may still depend on $\mathbf{k}$. The term with C_1 is the growing adiabatic mode, the term with C_2 is the decaying adiabatic one. The existence of these exact solutions directly follows from the observation made in [18] (see also the detailed explanation in [9]) that there always exists a solution for scalar perturbations in the flat ($\mathcal{K} = 0$) FRW Universe which has the following asymptotic behaviour in the synchronous gauge in the limit $k \to 0$ in terms of the Lifshitz variables: $\mu(\mathbf{k}) = 3h(\mathbf{k})$, $\lambda(\mathbf{k}) = 0$, $\delta\phi_j(\mathbf{k}) = 0$ (with no dependence on t) *irrespective of the structure and the properties of the energy-momentum tensor of matter.* Knowledge of the solutions (10,11) is not, however, sufficient to find the amplitude of generated perturbations if the number of scalar fields $N > 1$, in that case we have to integrate the system (7-9) completely in the limit $k \to 0$ at the inflationary stage.

Let us now consider a multiple inflationary stage with $\tilde{N}$ background scalar fields being in the slow-rolling regime ($\tilde{N} \leq N$), $\tilde{N}$ may depend on time. The energy density of all other scalar fields not being in the slow-rolling regime decreases exponentially with time and soon becomes negligible, thus, these fields should be simply omitted from the background equation (3). Then (3,4) simplify to

$$H^2 = \sum_{j=1}^{\tilde{N}} \frac{8\pi G}{3} V_j(\phi_j), \tag{12}$$

$$3H\dot{\phi}_j + V'(\phi_j) = 0, \qquad j = 1, .., \tilde{N}. \tag{13}$$

Now, for $k \ll aH$, the system (7-9) can be solved in a way completely analogous to [1, 9]. First, its solutions corresponding to growing adiabatic and non-decreasing isocurvature modes weakly depends on time, so for them Eqs. (8,9) take the form:

$$\Phi = \frac{4\pi G}{H} \sum_{j=1}^{\tilde{N}} \dot{\phi}_j \delta\phi_j, \tag{14}$$

$$3H\delta\dot{\phi}_j + V_j''\delta\phi_j = -2V_j'\Phi, \qquad j = 1, .., \tilde{N}. \tag{15}$$

The general solution is

$$\Phi = -C_1 \frac{\dot{H}}{H^2} - H\frac{d}{dt}\Big(\frac{\sum_j d_j V_j}{\sum_j V_j}\Big), \tag{16}$$

$$\frac{\delta\phi_i}{\dot\phi_i} = \frac{C_1}{H} - 2H\Big(\frac{\sum_j d_j V_j}{\sum_j V_j} - d_i\Big), \quad i,j = 1,..,\tilde N. \tag{17}$$

Here C_1 and d_j are integration constants, only $\tilde N - 1$ out of the $\tilde N$ coefficients d_j are linearly independent, and we will further use this freedom to add a constant term to them. The background quantities $H(t), \phi_j(t)$ are exact solutions of Eqs. (12,13). The mode with the coefficient C_1 is the growing adiabatic mode as can be seen from the comparison with (10,11), the other $\tilde N - 1$ modes are the non-decreasing isocurvature modes.

The expression for the decaying adiabatic mode immediately follows from the general expressions (10,11) which take the following form at the inflationary stage:

$$\Phi = C_2 \frac{H}{a}, \qquad \frac{\delta\phi_j}{\dot\phi_j} = -\frac{C_2}{a} \tag{18}$$

for all scalar fields (including those which are not in the slow-rolling regime). The expression for decaying isocurvature modes of slowly-rolling scalar fields may be found, similarly to [9], by assuming that all quantities in Eqs. (7-9) are proportional to $a^{-3}(t)$ multiplied by slowly varying functions of t (note that the approximate form (14,15) of these equations cannot be used now). The answer is

$$\Phi = \Psi = 0, \quad \delta\phi_j = \frac{\tilde d_j}{\dot\phi_j H^2 a^3}, \quad \sum_j \tilde d_j = 0, \quad j = 1,..,\tilde N \tag{19}$$

which may be easily verified by direct substitution using (12,13). Finally, all other $2(N - \tilde N)$ scalar modes connected with non-slowly-rolling scalar fields are decreasing isocurvature ones, too. We shall consider them below in connection with matching to a post-inflationary era.

Let us return to the most interesting non-decreasing modes. Another quantity which is useful for their description is the fractional comoving energy perturbation in each scalar field component

$$\Delta_j \equiv \frac{\delta\varepsilon_j^{(c)}}{(\varepsilon+p)_j} = \frac{\dot\phi_j\delta\dot\phi_j + V_j'\delta\phi_j + 3H\dot\phi_j\delta\phi_j - \dot\phi_j^2\Phi}{(\varepsilon+p)_j} = \frac{\partial}{\partial t}\left(\frac{\delta\phi_j}{\dot\phi_j}\right) - \Phi \tag{20}$$

where $\delta\varepsilon_j^{(c)}$ coincides with Bardeen's gauge-invariant quantity ϵ_m times the background energy density in the case of one scalar field (see [17]) and

relation (7) is used. Note also the following consequence of Eqs. (7-9) which is actually the Newton-Poisson equation in the cosmological case:

$$k^2\Phi = -4\pi G a^2 \sum_{j=1}^{N} \delta\varepsilon_j^{(c)} \ . \tag{21}$$

In the long-wave limit $k \to 0$, the substitution of expressions (10,11) into (20) gives 0. This means that the small-k expansion of Δ_j contains an additional k^2 multiplier in the case of both adiabatic modes, i.e., $|\Delta_j| \ll |\Phi|$ for them in this limit. On the other hand, $|\Delta_j|$ can be of the order of, and even much bigger than, $|\Phi|$ for isocurvature modes though the total comoving density perturbation $\sum_{j=1}^{N}\delta\varepsilon_j^{(c)}$ still contains the additional k^2 multiplier compared to Φ as follows from (21). Substituting the expressions (16,17) valid during the multiple inflationary stage into (20) we get:

$$\Delta_i = 2d_i\dot{H} + 8\pi G\sum_j d_j\dot{\phi}_j^2, \quad j = 1,..,\tilde{N}. \tag{22}$$

The next step is to determine the coefficients $C_1,\ d_j$ from amplitudes of quantum fluctuations of scalar fields generated during the inflationary stage. First, we invert (16,17) to obtain:

$$C_1 = \frac{8\pi G}{3H}\sum_j \frac{V_j}{\dot{\phi}_j}\delta\phi_j = -8\pi G\sum_j \frac{V_j}{V_j'}\delta\phi_j \ , \tag{23}$$

$$d_i = \frac{\delta\phi_i}{2H\dot{\phi}_i} - \frac{C_1}{2H^2} + \frac{\sum_j d_j V_j}{\sum_j V_j} \ , \quad i,j = 1,..,\tilde{N}. \tag{24}$$

Further, using the above-mentioned possibility to add the same constant to all d_j, we omit the last two terms in Eq. (24). Then all $\delta\phi_j$ in the r.h.s. of (23,24) have to be matched with quantum fluctuations of the scalar fields generated during the inflationary stage (we remind that this is a genuine quantum-gravitational effect). For all scalar fields being in the slow-rolling regime, $|m_{j,eff}^2| \equiv |V_j''| \ll H^2$, therefore, all mass- and Φ- dependent terms in Eqs. (8-9) may be neglected for $k \geq aH$, and even in the region $k < aH$ but $(t-t_k)H(t_k) \ll H^2(t_k)/|\dot{H}(t_k)|$ where t_k is the time when a mode k crosses the Hubble radius during the inflationary stage, i.e., $k = a(t_k)H(t_k)$ (it is in the latter region where the exact matching is performed). Then the $\delta\phi_j$'s behave like massless uncoupled scalar fields in the de Sitter background. The

standard quantization gives the well-known result (see e.g. [18]): for $k \ll aH$, the Fourier components of the fields are time independent ("frozen") and may be represented in the following form:

$$\delta\phi_j(\mathbf{k}) = \frac{H(t_k)}{\sqrt{2k^3}} e_j(\mathbf{k}) \tag{25}$$

where $e_j(\mathbf{k})$ are classical stochastic Gaussian quantities with vanishing average values $< e_j(\mathbf{k}) >= 0$ and the correlation matrix $< e_j(\mathbf{k}) e^*_{j'}(\mathbf{k}') >= \delta_{jj'}\delta^{(3)}(\mathbf{k}-\mathbf{k}')$. Note however that in the case of multiple inflation there may exist small effects [19] for which the approximation (25) is not sufficient and one has to take into account a small quantum correction to it reflecting the fact that the generated fluctuations are in a squeezed pure quantum state with a large but finite squeezing parameter r (the limit $r \to \infty$ is completely equivalent to (25)). As for scalar fields with large effective masses which are not in a slow-rolling regime, their fluctuations are negligible (apart from the case when they experience a non-equilibrium first-order phase transition during inflation which we do not consider here).

By substituting (25) into (23,24), we get finally (we denote by $C_1^2(k)$ the power spectrum of the stochastic quantity $C_1(\mathbf{k})$ and use a similar notation for all stochastic variables, $\langle f(\mathbf{k}) f^*(\mathbf{k}') \rangle = f^2(k)\delta^{(3)}(\mathbf{k}-\mathbf{k}')$):

$$\begin{aligned} C_1(\mathbf{k}) &= -\frac{8\pi GH}{\sqrt{2k^3}} \sum_j \frac{V_j}{V'_j} e_j \ , \quad C_1^2(k) = \frac{32\pi^2 G^2 H^2}{k^3} \sum_j \frac{V_j^2}{V_j'^2} \ , & (26) \\ d_i(\mathbf{k}) &= -\frac{3H}{2\sqrt{2k^3} V'_i} e_i \ , \quad d_i^2(k) = \frac{9H^2}{8k^3 V_i'^2} \ , \quad i,j = 1,..,\tilde{N} & (27) \end{aligned}$$

where all the time-dependent quantities in the r.h.s. are taken at $t = t_k$. The result for $C_1^2(k)$ coincides with that previously obtained in [1]. In the case of two scalar fields ($j = 1, 2$), we reproduce the results of ref. [9] where the notation $C_3 \equiv d_1 - d_2$ was used. The expressions (16,17,26,27) are the main results of this section.

3 Matching to a post-inflationary era

As was mentioned in the introduction, in the case of isocurvature modes we don't have general expressions like (10,11) for adiabatic modes, hence the

post-inflationary behaviour of isocurvature perturbations is not universal and depends on additional assumptions. In particular, there could be no such perturbations at all soon after the end of an inflationary stage. Therefore we will consider further a number of specific models in which they may be present even nowadays. The most natural way to achieve it is to assume that one of the inflaton scalar fields remains uncoupled from usual matter (baryons, photons, etc.) all the time since the end of the multiple inflationary stage up to the present moment, and that its particles or products of their decay (still uncoupled from usual matter) constitute today a part of the cold dark matter with a dust-like equation of state ($p \ll \varepsilon$).

The non-decreasing mode of isocurvature fluctuations in a system of two uncoupled components consisting of dust-like matter on one hand, and radiation coupled to baryons on the other hand, can be characterized by a fractional comoving energy density perturbation in the dust-like component

$$\delta_m \equiv \frac{\delta\varepsilon_m^{(c)}}{\varepsilon_m} = \delta_i \frac{\Omega_i}{\Omega_m} \ , \qquad \delta_i \equiv \frac{\delta\varepsilon_i^{(c)}}{\varepsilon_i} \approx \Delta_i \ , \tag{28}$$

that remains constant during the radiation-dominated era (see, e.g., [20]-[21] for reviews). Here Ω_i is the present-day density (in terms of the critical one) of that part of cold dark matter which is the relic from the inflationary era while Ω_m refers to all the cold dark matter ($\Omega_i \leq \Omega_m$). Note that we have for the number density perturbation, $\frac{\delta n_i}{n_i} = \delta_i$ where n_i is the number density of relics. $\Omega_{tot} = \Omega_m + \Omega_{bar} = 1$ with great accuracy for cosmological models having an inflationary stage (the energy density of the cosmological term, if non-zero, should be added to Ω_{tot}, too). After the transition to the matter-dominated stage at redshifts $z \approx 10^4$, this mode produces a growing adiabatic mode of fluctuations which evolves, as usually, $\propto a(t)$ afterwards.

Therefore, we have to relate δ_i at the radiation-dominated stage with Δ_i at the inflationary stage, as given in Eq. (22). We further specialize to the case of two inflaton scalar fields and replace the subscripts 1, resp. 2 by h (heavy), resp. l (light). Then Eqs. (16,17,22) take the following form which generalizes the results of ref. [9]:

$$\Phi(t) = -C_1(\mathbf{k})\frac{\dot{H}}{H^2} + \frac{C_3(\mathbf{k})}{3}\frac{V_l V_h'^2 - V_h V_l'^2}{(V_h+V_l)^2} \ , \tag{29}$$

$$\frac{\delta\phi_h}{\dot{\phi}_h}(t) = \frac{C_1(\mathbf{k})}{H} + 2C_3(\mathbf{k})\ \frac{HV_l}{V_h+V_l} \ , \tag{30}$$

$$
\begin{aligned}
\frac{\delta\phi_l}{\dot{\phi}_l}(t) &= \frac{C_1(\mathbf{k})}{H} - 2C_3(\mathbf{k})\,\frac{HV_h}{V_h+V_l}\,, && (31)\\
\Delta_h(t) &= -\frac{C_3(\mathbf{k})}{3}\,\frac{V_l'^2}{V_h+V_l}\,, && (32)\\
\Delta_l(t) &= \frac{C_3(\mathbf{k})}{3}\,\frac{V_h'^2}{V_h+V_l} && (33)
\end{aligned}
$$

where C_1 and $C_3 = d_h - d_l$ are given in Eqs. (26,27). Two essentially different cases may take place which we call the cases of heavy relics and light relics.

1. Heavy relics.

This case arises when the inflaton field that remains uncoupled from usual matter after inflation has an effective mass larger than H at the end of inflation. Then this "heavy" scalar field ϕ_h is in the slow-rolling regime in the first part of inflation, but it goes out of this regime when H becomes less than the effective mass during inflation. Somewhat earlier, its energy density becomes much smaller than the total one. Let us take the simplest case where the effective mass is constant, so that the potential is $V_h = m_h^2\phi_h^2/2$, and $G\phi_h^2 \gg 1$ at the early stages of inflation (for the field to be in the slow-rolling regime initially). Note for completeness that it is not possible to realize such a scenario for a steeper power-law potential V_h, in particular for $V_h = \lambda\phi_h^4/4$, because then the effective mass remains smaller than H till the end of inflation.

If $\varepsilon_h \ll \varepsilon_{tot}$, then irrespective of the fact whether the field ϕ_h is in the slow-rolling regime or not, the right-hand side of Eq.(9) may be neglected for isocurvature modes. Now we need to solve this equation in the limit $k^2 = 0$. Note that then the left-hand sides of Eqs.(9) and (4) coincide in the case of a massive scalar field without self-interaction. So, one of the solutions is $\delta\phi_h \propto \phi_h(t)$ where $\phi_h(t)$ is the exact solution of Eq.(4) in a background driven by the other scalar field through Eq.(3). The other linearly independent solution can be found from the Wronskian condition but we don't need it here because, if ϕ_h is still in the slow-rolling regime, Eq.(30) may be applied which reads $\delta\phi_h = 2C_3H\dot{\phi}_h = -2C_3m_h^2\phi_h/3$ for $V_h \ll V_l$. Now we use the constancy of the quantity $\delta\phi_h(t)/\phi_h(t)$ during a transition from inflation to the radiation-dominated stage. For $m_ht \gg 1$ at the latter stage, the heavy field is in the WKB regime of oscillations with the frequency m_h (see, e.g.,

[9] for exact expressions). Averaging over the oscillations, we obtain

$$\delta_h = 2\,\frac{\delta\phi_h}{\phi_h} = -\frac{4}{3} m_h^2 C_3(\mathbf{k}) \ . \tag{34}$$

Note that though Eq. (34) looks like Eq. (19.18) in [20], it is not exactly the same because it refers to a different quantity (a comoving energy perturbation vs. an energy perturbation in the longitudinal gauge (6)), and we apply it in a different regime (at the radiation-dominated vs. the inflationary stage). From (27), the amplitude of the fractional energy and number density perturbation in the relic dust component during the radiation-dominated stage follows:

$$k^3 \delta_h^2(k) = 2H^2(t_k) \left(\frac{1}{\phi_h^2} + \frac{m_h^4}{V_l'^2} \right)_{t_k} \tag{35}$$

where the first term inside the brackets in the r.h.s. of the last equation should be omittted if $H(t_k) < m_h$. The power spectrum has a slope $n \approx -3$ similar to that of $\Phi^2(k)$ for adiabatic perturbations.

To make a quantitative comparison between contributions of isocurvature and adiabatic modes to effects observable today, one should take into account that, due to the properties of the transfer function for isocurvature fluctuations in the CDM+radiation model, an isocurvature density fluctuation δ_m at the radiation-dominated stage produces the same adiabatic mode after transition to the matter-dominated stage ($a(t) \propto t^{2/3}$) as the initial adiabatic mode with $\Phi = \delta_m/5$ for scales exceeding by far the present comoving scale corresponding to the cosmological horizon at the moment of matter-radiation equality $R_{eq} \approx 30h^{-1}Mpc$, $H_0 \equiv h \cdot 100\ km\ s^{-1} Mpc^{-1}$ (Ω_{bar} is assumed to be small, too). For $kR_{eq} > 1$, the equivalent amplitude of Φ is even less. On the other hand, isocurvature fluctuations produce 6 times larger angular temperature fluctuations $\Delta T/T$ in the CMB at angles $\theta > 30'$ for the same amplitude of long-wave density perturbations at the matter-dominated stage, i.e., $\Delta T/T = 2\delta_m/5$ vs. $\Delta T/T = \Phi/3$ for adiabatic perturbations [22], [23], [24] (see also [20]-[21] for reviews). Due to the latter reason, it has been long known that isocurvature fluctuations with a flat ($n = -3$) initial spectrum cannot be responsible for the observed large-scale structure and $\Delta T/T$ fluctuations in the Universe.

For a power-law V_l with the last part of inflation driven by the light scalar field, $V_h' > V_l'$ in the region $V_h \sim V_l$ where the transition from heavy to light

scalar field domination of the total energy takes place. Then the second term inside the brackets in (35) is the dominant one, while still much smaller than $\Phi^2(k) = 9C_1^2(k)/25$. Therefore isocurvature fluctuations, if present, are less than adiabadic ones in double inflationary models in the region around the break in the perturbation spectrum due to a transition between the two phases of inflation, and their possible presence changes nothing regarding the confrontation of these models with observational data (see, e.g. [12]). But on much smaller scales, when the first term inside the brackets in (35) is dominant, isocurvature perturbations become much larger. In that case

$$k^3\delta_h^2(k) = \frac{2H^2}{\phi_h^2}(t_k) \ . \tag{36}$$

Alternatively, this result may be obtained very simply by considering the heavy field as a test field in the de Sitter background and using the expressions (25,34). The spectrum (36) grows with k (because ϕ_h quickly decreases with t) until the point $H(t_k) \sim m_h$ is reached, after that it falls abruptly.

If, e.g., $V_l = \frac{1}{2}m_l^2\phi_l^2$ and $m_l \ll m_h$, then, using Eqs. (2.10-2.15) of ref. [9] and Eq. (35), we get

$$k^3\delta_h^2(k) = \frac{8\pi G m_l^2}{3}\left(\left(\frac{s_0}{s_0 - \ln\frac{k}{k_b}}\right)^{m_h^2/m_l^2} + \left(\frac{m_h}{m_l}\right)^4\right), \quad k \gg k_b \tag{37}$$

where k_b is the location of the break and $s_0 \gg 1$ is the number of e-folds during the second phase of inflation driven by the light scalar field ($s_0 \approx 60$ to account for observational data, see [12]). The expression (37) is derived under the approximation $m_h^2/m_l^2 < s_0$ which corresponds to the absence of a power-law intermediate stage between the two phases of inflation (double inflation without break, according to the terminology of [9]), a more suitable condition perhaps is the absence of oscillations or a smooth transition in the spectrum, which will be the case for $m_h/m_l < 15$ [25] or $m_h^2/m_l^2 < 4s_0$. In the opposite case of double inflation with a break, there is no growth of isocurvature fluctuations at small scales. Note that the effect of growth in the isocurvature perturbation spectrum was previously noticed from numerical calculations for a similar model in [6].

For not too small scales when $\ln(k/k_b) \ll s_0$, the first term in the spectrum (37) is power-law like with a small exponent: $k^3\delta_h^2(k) \propto (k/k_b)^\alpha$, $\alpha =$

$m_h^2/m_l^2 s_0$. The perturbations reach their maximum on short scales for which $s_0 - \ln(k/k_b) \sim m_h^2/m_l^2$, due to the disappearance of the first term in (37) for $H(t_k) < m_h$, its value being

$$\left(k^3 \delta_h^2(k)\right)_{max} \sim G m_l^2 \left(\frac{m_l^2 s_0}{m_h^2}\right)^{m_h^2/m_l^2} . \tag{38}$$

It is interesting that for $\sqrt{G} m_l \sim 10^{-6},\ s_0 \sim 60$ the maximal value of the quantity (38) as a function of m_h/m_l, though still smaller than unity, is not far from it (it is reached for $m_h/m_l \approx 5$). Hence, such a model can be used to produce a significant number of primordial black holes with rather small masses (for a review of observational upper limits on the number density of PBHs see, e.g., [26]-[27]). Then, however, it cannot explain the observed large-scale structure and $\Delta T/T$ fluctuations in the Universe because m_h/m_l is required to be $\approx 12 - 14$ (and certainly more than 8) for this aim, see [12].

2. Light relics

In this case, $m_{eff}^2 \ll H^2$ for one of the inflaton scalar fields during the whole inflation. To avoid this "light" field to be dominating during the last part of inflation and after its end, we have to assume that its energy density $\varepsilon_l \ll \varepsilon_{tot}$ during inflation, too. Then we have from Eq. (31):

$$\delta\phi_l = -2C_3 H \dot{\phi}_l = \frac{2}{3} C_3 V_l' . \tag{39}$$

If C_3 from Eq. (27) is substituted into this expression and it is assumed that $V_l' \ll V_h'$, the standard expression (25) for fluctuations of a test scalar field on a de Sitter background arises once more. Let $V_l = \frac{1}{2} m_l^2 \phi_l^2$, then we may repeat the derivation made in the previous subsection to get the expression (36) (with the index "h" changed to "l") for the fractional density perturbation in the light relic component. Now ϕ_l is practically constant during inflation (and less than $G^{-1/2}$ to avoid a second inflationary phase), so the spectrum is falling with k. Isocurvature perturbations may be larger than adiabatic ones if ϕ_l is small enough, but this does not lead to interesting cosmological models for a smooth potential V_h satisfying the slow-rolling conditions due to the reason mentioned in the previous subsection in connection with the $n = -3$ initial isocurvature perturbation spectrum.

3. Intermediate relics

Let us briefly consider the case of an inflaton field which remains uncoupled from usual matter after inflation, dominates during the first phase of inflation (so we call it "heavy") and has the quartic potential $V_h = \lambda_h \phi_h^4/4$. During the last part of inflation, $s < s_0$, where s is the number of e-folds measured from the end of inflation and $s_0 \gg 1$ is the moment when $V_h = V_l$, $\varepsilon_h \ll \varepsilon_{tot}$, however $m_{h,eff}^2 \equiv 3\lambda_h \phi_h^2 \ll H^2$ (and less than the effective mass of the other scalar field, too). So, this initially "heavy" inflaton becomes "light" in the last part of inflation. That is why we call this case the intermediate one.

Then, for $s \ll s_0$, $\delta\phi_h = 2C_3 H \dot{\phi}_h = -2C_3 \lambda_h \phi_h^3(t)/3$ as in the case of massive relics. Here the quantity $\delta\phi_h/\phi_h$ is not constant during the last period of inflation and the transition to the radiation-dominated stage. Therefore, an exact matching (as it was done in the subsection 1) is not possible, but we may make a matching by order of magnitude using the fact that δ_h at the radiation-dominated stage is of the order of $\delta\phi_h/\phi_h$ at the end of inflation. Using (30,34), we get: $\delta_h^2(k) = const \cdot C_3^2(k) m_{h,eff}^4(t_f)$ where t_f is the moment when inflation ends and $const = \mathcal{O}(1)$.

a) $V_l = m_l^2 \phi_l^2/2$, $\lambda_h \gg G m_l^2$.
Then $\phi_h^2 = m_l^2/\lambda_h \ln(s_0/s)$ during the last period of inflation. Therefore, $m_{h,eff}^2(t_f) = 3m_l^2/\ln s_0$, and we arrive at the following result

$$k^3 \delta_h^2(k) = const \cdot H^2(t_k) \left(\frac{1}{\lambda_h^2 \phi_h^6} + \frac{1}{m_l^4 \phi_l^2} \right)_{t_k} \frac{m_l^4}{\ln^2 s_0} \tag{40}$$

where $const = \mathcal{O}(1)$. In particular, for $s \ll s_0$,

$$k^3 \delta_h^2(k) = const \cdot \frac{\lambda_h s(k) \ln^3 (s_0/s(k))}{\ln^2 s_0} \tag{41}$$

where $s(k) \equiv s(t_k) = \ln(k_f/k) \gg 1$ and $k_f = a(t_f)H(t_f)$. The spectrum is approximately flat. Though it has a smooth maximum at $s = s_0 e^{-3}$, this maximum is not as strongly pronounced as in the case of massive heavy relics. The amplitude grows $\propto (k/k_b)^{1.5}$ for $s_0 - s \ll s_0$ where k_b is the inverse comoving scale corresponding to the first horizon crossing at the moment s_0.

b) $V_l = \lambda_l \phi_l^4/4$, $\lambda_h \gg \lambda_l$.
Now $\phi_h^2 = \lambda_l \phi_l^2 / 2\lambda_h \ln(s_0/s)$ during the second phase of inflation. Thus, $m_{h,eff}^2(t_f) \sim \lambda_l / G \ln s_0$ and

$$k^3 \delta_h^2(k) = const \cdot H^2(t_k) \left(\frac{1}{\lambda_h^2 \phi_h^6} + \frac{1}{\lambda_l^2 \phi_l^6} \right)_{t_k} \frac{\lambda_l^2}{G^2 \ln^2 s_0} \tag{42}$$

where $const = \mathcal{O}(1)$. In particular, for $s \ll s_0$,

$$k^3 \delta_h^2(k) = const \cdot \frac{\lambda_h \ln^3 (s_0/s(k))}{s(k) \ln^2 s_0} \ . \tag{43}$$

The spectrum is approximately flat and grows slightly towards large k's. Once more, its amplitude grows $\propto (k/k_b)^{1.5}$ for $s_0 - s \ll s_0$.

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